Written Exam for the M.Sc. in Economics autumn 2011-2012

Contract Theory

Final Exam / Master's Course

January 6, 2012

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Attempt both questions

Question 1 (adverse selection)

The following is a model of an insurance market with adverse selection. It builds on the standard adverse selection model that we studied in the course.

The principal (P) is a monopoly insurance company and the agent (A) is a car owner who may want to take a car insurance. Depending on how skillful A is as a driver, she may or may not have an accident. The probability of having an accident depends on A's type. A skillful (and therefore a *low*-demand) driver has an accident with probability $\underline{\theta}$, and a less skillful (and therefore a *high*-demand) driver has an accident with probability $\overline{\theta}$. Assume that $0 < \underline{\theta} < \overline{\theta} < 1$.

A's disutility of having an accident, measured in monetary terms as a deduction from her income, is denoted $d > 0$, and A's monetary income is denoted $w > d$. Moreover, A's payment *to* P in case there is no accident is denoted p; and the *net* compensation A *receives* from P in case there indeed is an accident is denoted a. A is risk averse and her utility function is denoted u (where $u' > 0$ and $u'' < 0$). Therefore, A's utility if taking the insurance is

$$\begin{cases} u(w-d+a) & \text{if having an accident} \\ u(w-p) & \text{if not having an accident.} \end{cases}$$

P is risk neutral and wants to maximize its expected profits. It does not know the type of A, but assigns the probability $\upsilon \in (0,1)$ to the event that $\theta = \underline{\theta}$.

P offers a menu of two distinct contracts to A. As in the course, the contract variables are indicated either with "upper-bars" or "lower-bars", depending on which type the contract is aimed at. The contract variables are p and a. However, to solve the problem it is more convenient to think of P as choosing the utility levels directly, instead of the contract variables. Thus introduce the following notation:

$$\overline{u}_N \equiv u(w-\overline{p}), \quad \overline{u}_A \equiv u(w-d+\overline{a}), \quad \underline{u}_N \equiv u\left(w-\underline{p}\right), \quad \underline{u}_A \equiv u(w-d+\underline{a}).$$

Also let h be the inverse of u (hence $h' > 0$ and $h'' > 0$). We can now rewrite the problem as follows. Given that P is risk neutral and wants to maximize its expected profit, P's objective function can be written as

$$\begin{aligned} V &= \upsilon\left[(1-\underline{\theta})\,\underline{p} - \underline{\theta}\underline{a}\right] + (1-\upsilon)\left[\left(1-\overline{\theta}\right)\overline{p} - \overline{\theta}\overline{a}\right] \\ &= \upsilon\left[w - \underline{\theta}d - (1-\underline{\theta})\,h(\underline{u}_N) - \underline{\theta}h(\underline{u}_A)\right] \\ &\quad + (1-\upsilon)\left[w - \overline{\theta}d - \left(1-\overline{\theta}\right)h(\overline{u}_N) - \overline{\theta}h(\overline{u}_A)\right]. \end{aligned}$$

P wants to maximize V w.r.t. $(\underline{u}_N, \underline{u}_A, \overline{u}_N, \overline{u}_A)$, subject to the following four constraints:

$$\left(1-\overline{\theta}\right)\overline{u}_N + \overline{\theta}\overline{u}_A \geq \overline{U}^*, \tag{IR-high}$$

$$(1-\underline{\theta})\,\underline{u}_N + \underline{\theta}\underline{u}_A \geq \underline{U}^*, \tag{IR-low}$$

$$\left(1-\overline{\theta}\right)\overline{u}_N + \overline{\theta}\overline{u}_A \geq \left(1-\overline{\theta}\right)\underline{u}_N + \overline{\theta}\underline{u}_A, \tag{IC-high}$$

$$(1-\underline{\theta})\,\underline{u}_N + \underline{\theta}\underline{u}_A \geq (1-\underline{\theta})\,\overline{u}_N + \underline{\theta}\overline{u}_A, \tag{IC-low}$$

where

$$\overline{U}^* \equiv \left(1-\overline{\theta}\right)u(w) + \overline{\theta}u(w-d), \quad \underline{U}^* \equiv (1-\underline{\theta})\,u(w) + \underline{\theta}u(w-d)$$

are the two types' outside options.

a) At the first-best optimum (i.e., the optimum when A's type is observable), both types are offered a contract with full insurance (so that $\overline{u}_N = \overline{u}_A$ and $\underline{u}_N = \underline{u}_A$). Explain, in words, the economic logic behind this result.

b) Show that the constraints (IC-high) and (IC-low) jointly imply that $\underline{u}_N - \underline{u}_A \geq \overline{u}_N - \overline{u}_A$.

c) Assume that the constraints (IR-high) and (IC-low) are lax at the second-best optimum (so that they can be disregarded). Show that, at the second-best optimum, the high type is fully insured ($\overline{u}_N = \overline{u}_A$) whereas the low-type is underinsured ($\underline{u}_N > \underline{u}_A$).

d) In some other adverse selection models that we studied, the outside option for the "good" type was (sufficiently much) more attractive than the "bad" type's outside option. This gave rise to a phenomenon called "countervailing incentives". Answer, in words, the following questions: (i) What is by meant by "countervailing incentives"? (ii) What are the possible consequences of this phenomenon in terms of efficiency and rent extraction at the second-best optimum? (iii) What is the intuition for the results under (ii)?

Question 2 (moral hazard)

This is a model of so-called sharecropping. It is identical to one that we studied in the course.

A landlord (the principal, P) owns a piece of land and wants to lease the land to a poor farmer (the agent, A). If entering such an agreement, A can, when farming the land, choose whether to work hard ($e = 1$) and incur a cost $\psi > 0$, or not to work hard ($e = 0$) and incur no cost. Depending on whether A works hard or not and on the weather, the output that is produced may be high ($q = \overline{q}$) or low ($q = \underline{q}$, with $0 \leq \underline{q} < \overline{q}$). The probability with which the output is high equals π_1 if A works hard and π_0 if A does not work hard. Assume that $0 < \pi_0 < \pi_1 < 1$. The market price of the output equals unity. Therefore, q is also the market value of the output.

P (and the court) can observe which output that is realized ($\overline{q}$ or $\underline{q}$) but not whether A has worked hard or not. Therefore, in principle, the contract between P and A could consist of *two* numbers, indicating how much A should pay P in each state. However, the contract that is actually used is a so-called sharecropping contract, which is characterized by a *single* number, $\alpha \in [0, 1]$. The number α is the share of output that A is allowed to *keep*, whereas the remaining share $1 - \alpha$ is paid to P.

Therefore, P's expected profit equals

$$V_e = (1 - \alpha) \left[\pi_e \overline{q} + (1 - \pi_e) \underline{q}\right] \qquad \text{for } e \in \{0, 1\}.$$

Moreover, A's expected utility equals $U_1 = \alpha \left[\pi_1 \overline{q} + (1 - \pi_1) \underline{q}\right] - \psi$ if working hard and it equals $U_0 = \alpha \left[\pi_0 \overline{q} + (1 - \pi_0) \underline{q}\right]$ if not working hard. A's outside option would yield the utility zero. A is protected by limited liability, meaning that a contract cannot stipulate that A must pay, in net terms, some amount of money to P. It is assumed that P has all the bargaining power and makes a take-it-or-leave-it offer to A.

a) Explain why, given the assumed contract form and the assumption that $\alpha \in [0,1]$, the limited liability constraint is automatically satisfied.

b) Suppose P does *not* want to induce A to work hard. Formulate P's optimization problem in this situation, solve the problem, and show that P's expected profit at the optimum equals $V_0^* = \pi_0 \overline{q} + (1 - \pi_0)\, \underline{q}$.

c) Suppose P *does* want to induce A to work hard. Formulate P's optimization problem in this situation, solve the problem, and show that P's expected profit at the optimum equals

$$V_1^* = \left[\pi_1 \overline{q} + (1 - \pi_1)\, \underline{q}\right] - \frac{\left[\pi_1 \overline{q} + (1 - \pi_1)\, \underline{q}\right] \psi}{(\pi_1 - \pi_0)\left(\overline{q} - \underline{q}\right)}.$$

d) Explain, in words, in what sense the sharecropping contract form gives rise to underprovision of effort relative to *both* the second best optimum (i.e., the optimum given unobservable effort and a contract with two numbers) *and* relative to the first best optimum (i.e., the optimum given observable effort and a contract with two numbers). Also explain the intuition for (i.e., the logic behind) each one of those two results.

END OF EXAM